Exercise 224

The depth (in feet) of water at a dock changes with the rise and fall of tides. It is modeled by the function $D(t) = 5\sin\left(\dfrac{\pi}{6}t - \dfrac{7\pi}{6}\right) + 8$, where t is the number of hours after midnight. Determine the first time after midnight when the depth is 11.75 ft.
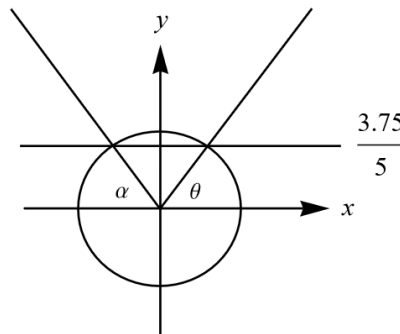
Solution

Set $D(t) = 11.75$ and solve the equation for t.

$$11.75 = 5\sin\left(\frac{\pi}{6}t - \frac{7\pi}{6}\right) + 8$$

$$3.75 = 5\sin\left(\frac{\pi}{6}t - \frac{7\pi}{6}\right)$$

$$\frac{3.75}{5} = \sin\left(\frac{\pi}{6}t - \frac{7\pi}{6}\right)$$

There are two angles, α and θ, that give $3.75/5$ after taking the sine. Taking the arcsine of $3.75/5$ on the calculator gives θ. α is $\pi - \theta$.

$$\alpha = \pi - \sin^{-1}\left(\frac{3.75}{5}\right) = \frac{\pi}{6}t - \frac{7\pi}{6} \quad\text{or}\quad \theta = \sin^{-1}\left(\frac{3.75}{5}\right) = \frac{\pi}{6}t - \frac{7\pi}{6}$$

$$\frac{7\pi}{6} + \pi - \sin^{-1}\left(\frac{3.75}{5}\right) = \frac{\pi}{6}t \quad\text{or}\quad \sin^{-1}\left(\frac{3.75}{5}\right) + \frac{7\pi}{6} = \frac{\pi}{6}t$$

$$\frac{6}{\pi}\left[\frac{7\pi}{6} + \pi - \sin^{-1}\left(\frac{3.75}{5}\right)\right] = t \quad\text{or}\quad \frac{6}{\pi}\left[\sin^{-1}\left(\frac{3.75}{5}\right) + \frac{7\pi}{6}\right] = t$$

$$11.38 \approx t \quad\text{or}\quad 8.62 \approx t$$

The first time after midnight that the depth is 11.75 ft is 8 hours and $0.62(60) \approx 37.2$ minutes.